# Mathematical Abstraction Theory, the Fundamentals for

Knowledge Representation and

Self-Evolving Autonomous Problem Solving Systems

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## Background

I would like to speak a little bit of the history in background of my study choice. As far as I can recall I have been interested of infinite, of some kind of “*quantum jumps*” which germinate and herald new inventions non inferable by using numerable proceedings. That led me by natural steps to the core of *mathematics with its abstract methods*. Later I became deeply interested in theoretical physics via physical implementations finding out that there will always be equations to be non- solvable by the means of numerated nature. I later invented some proper methods for obtaining arbitrarily accurate results for very general differential equations adjustable to computer programming, but felt it may still *be some automatic general way* encountering complex innumerably solvable puzzles *equating instant human invention processes*. While pondering about

*assistant depictions in geometrical problems* it came to my mind that *abstraction* used in free algebras could be a promising starting point. However first I should have to have coped with the question “*what actually is generally taking the definition of problem*”? It took some time to realize that it is merely a quadruple comprising syntax part joint in most general stile to semantics having distinct sections, namely 1. *entity representation*, 2. *memory* thereof parallel abstract-solutions linked to 3. *abstract level relations* in perceivable problem entities and 4. automata to *finally recognize* if the results obtained are proper regarding to the perceived environments.

Generally in the art there have been concentrations mainly on finite tree structures or sets of node- edge –rules in graphs and thereof ordering and computation time/halting problems studies *without touching generic problem solving* on those or more covering structures. Signs became to be seen of more ambitious stile in 1980s on perception targeting, hence rising up questions about conceptualization, but *only of implicit nature*, waiting for explicit formulations. In my study formalism is carried out fitting also to infinite proceedings, hence consenting handling *with function variables e.g. Schrödinger equation solutions in Quantum physics* and bit entities in chaos research area and in *unpredictable behavior phenomena* overall.

## Current Trends in Science overall

Conceptual graphs constitute equivalence classes as the form of elements in closed quotient systems, meaning that parallel transformations applied to those classes inevitably *drop images back into the set of those particular classes, which guarantees automated problem solving* and consequently is in the interest of this research. As an incalculably important contributing corollary this sprouts up consideration of autonomous *problem solving system evolution in ever deepening directions* deriving new approaches from lover level results. *As executions of the highlighted system* outside the evident robotics emerges quantum computing in the form of teleportation for new generalized information transportation.

Multiple order and dimension partially quotient algebras over triple (partition degree, fundamental abstraction relation set, quotient transducer set) can be seen manifesting entangled parallelism as conceptualism over set of algebras Tirri SI (2014), and classified closure systems by satisfying commutative property.

Every entity and processes *thereof are mathematically describable* and can be presented by appropriate realization of free abstract algebras. This is currently leading to interesting approaches to explain prevalent perception in physics: *Parallel information processes* are seen to explain *probability phenomena* comprising also its “annihilated” negative compound Feynman Richard P (1987) covering a variety of perceptions. Furthermore *information fluctuations are increasingly regarded equating with continuous* processes of tangible matters and *regarded as entropy* in cosmology studies e.g. about black hole horizon Hawking Steven W (2014) and energy teleportation Hotta Masahiro (2010, 2014), “Everything is Information ([*Participatory Anthropic*](http://discovermagazine.com/2002/jun/featuniverse#.UvxOUrTVdnA)[*Principle*](http://discovermagazine.com/2002/jun/featuniverse#.UvxOUrTVdnA))” Wheeler John A (1983, experimental proof Jacques V. et al (2007)).

Conceptual based information representation regardless of the complexity rate is comprehensible by simple algorithm reading learning systems like Google Deep Learning or link-recognizing IBM´s AI “Watson”. Thus Artificial Generic Intelligence is possible to emerge *without human assistance*, if the magnitude of available information is rich enough comprising also vital abstract mathematical fundamentals for wider relation structuring.

***Technical Part***

*First order problem objects*

* are ground perceptions the syntax of which are described by nets, *the linked entities in generalized free algebra*.
* Those syntax elements are thereafter dispersed to *conceptual parts with common origin* determined by normal form transducers of the same type, manifesting notional congruent (abstraction) classes in the set of the nets.

*Solution candidates for first order problem objects*

* are ground *transducers derived parallel with conceptual representative counterparts in memory* subject to abstraction relation between targets and on the other hand results, where *transducers are joint rewriting systems* embedded by nethomomorphisms.
* Achieved candidates are checked by automata (e.g. regular net homomorphism recognizer over suited domains) appropriate to tasks in hands corresponding to *realizations of net algebra*. Accepted new solutions (having produced acceptable sets of nets) added to the set of already known ones expand the solving power in the forthcoming - hereby setting a base for self-evolving autonomous learning systems, manifesting an appropriate closure system for iteration.

## Ingredients

* **Net** formalism and consequent substitution in rewritings allow loop structure manipulations, essential as realizations for studying e.g. functions in brain configurations and in iterative calculus.
* Nets are giving a natural prosperous syntax ground for transformations, the semantic side for a variety of algebras constituting as realizations of free algebra elements unlimited numbers of in- and out gateways.
* Nets consist of letters in 3 different alphabets establishing 1. operators, 2. substitutable letters and 3. in-/out –positions, being derived inductively over those alphabets by tied gluing forms. The positions of enclosements in nets, essential to determine redexes, are determined by the arity letters in the gluing forms.
* The presentation for nets can also be expanded for multidimensional Cartesian cases. Nets as class representatives can be specifically chosen root-wise for needed purposes in hands.
* **Realization**: Semantic shifts from operator letters of nets in *generalized free universal algebra* to operations in *semantic algebra* are executed by *homomorphisms* joining each ranked letter to a function/relation in the set of desired algebra, taking account of up- streaming environments (coming parts in data detection) in nets. Free algebra operations as of unconstrained nature constitute important special examples of semantic algebra elements.
* **Net homomorphisms** are complex functions over the whole spectrum of the net structures being a generalization (for multiple out-arity cases) of tree homomorphism replacing *piecewise net operators to sets of nets* starting from the roots of any representatives of the applied net.
* **Substitutions** (free and environment sensitive) are special cases of net homomorphisms, set to replace position determined frontier letters with set of nets liable to conditions in RNS´s.
* Intervening **Renetting systems** (RNS´s), by normal forms abstracting perceptions to multilevel handling, thus yielding solving procedure for any conceivable problem, serve as an essential explicit mathematics to see over details, posing the ground in artificial generic intelligence as the practical tools in robotics.
* RNS´s for net transformations comprise as basic elements, rules suitable for rewriting loops by differentiating left and right *substitutions* for variables. Right side substitutions permit new link creations hence enabling to avoid using infinite number of rules.
* RNS´s confront the challenge of holistic method with redex-position definitions harnessing pin-pointed replacements in large environments delivering an explicit tool, not just an implicit transformer as node-edge rule sets.
* Limit demands in RNS´s constitute a set of prerequisite frames to transforming, comprising rule ordering or application orders for left and right side substitutions, requirements of characters in net applicants such as possessing certain nodes, linkages to itself or to the environment or encompassing options for resulting nets to be realized in specific algebras.
* RNS´s themselves are expanded to form net structures of themselves in a form of **Transducers** (TD´s), the realizations of net syntax as *RNS´s embedded into the nodes*. Special cases of transducers are e.g. generalized equations being represented as nets, the identifier equalized with binary operator and other elements as TD´s; genetic rewriting can be seen as transducers concentrating to horizontal views in leaves orderings.
* **Automata** for checking the semantic outcomes is postulated to determine inclusions in sets of algebra liable to manifested net homomorphism set to crawl in nets, selecting the nets

the realizations of which equate with prerequisites subject to the applications, e.g. coinciding with Boolean inference to true or untrue clauses.

It is to be notified that automata in its broader sense are able to be defined as transducer, setting prerequisites that acceptable nets have to intersect nets in the initialized final set of automata.

* + **Abstraction relation** is the embodiment of the set of net pairs implemented by intervening *RNS-normal forms*, the productions of elements with common origin, coequaling *perception and its notional part*. Transformation results from perceptions are derived by transducers, representatives of classes related to abstraction classes of parallel nets. This leads to producing all necessary transducer solutions establishing a closure structure as manifesting *abstract quotient algebra*, with transducer classes as operations.
  + Abstraction relations which are preserved in transformation processes are *classified by the sets of intervening RNS´s* comprising PRNS(partition); GPRNS(generalized partition with distinct nature); CLRNS(colouring) and UPRNS(environment sensitive universal partition), and theorem results about relations between the mentioned types are achieved. Colouring RNS´s with substance covers are proved to generate PRNS-related abstraction relation.
  + Environment sensitive universally partitioning UPRNS´s are taken into account to posess additional memorizing power *over the seeming circumstances* of targeted problems. Results show that the necessary *commutativity* in order to produce abstract net classes is also in this case possible to achieve thereby expanding considerably the solution space.

## Problem and Solution

* + One of the most demanding task to comprehend has been to defining context “*problem*” and thereof “*solution”*, I determine that to comprising *knowledge representation* positioned to undergo *conceptual processes* by intervening RNS normal forms, consequently *establishing abstract counterparts* in the set of *already known entities* e.g. in memory storage; further orchestrating *TD formation as solutions* to pose *confluence modulo net abstraction relation* constituting partially quotient algebra, net class representatives later *qualified by the*

*recognizers appropriate* to the realizations in hands. The resulting TD´s manifest *solid solving system closure* applicable for any semantic needs.

## RNS-generation

* Supervision inside net structures requires overlapping formalism and that is manifested by **NUO-presentations**. That construction illuminates block connections inside nets thereby enabling to construct *net block homomorphism* (NBH), a generalization of net homomorphism for a more expansive domain.
* *Alphabetical abstraction* **net block homomorphism** (AlpANBH) induces a new RNS-type able to *generate the earlier* presented RNS-types, and a new abstraction equivalence relation over the whole set of the nets to encountering perception conceptualization can be defined thus validating commutative property by each RNS-type. Finally it will be proven that *operational efficiency* of the set of all NBH´s and on the other hand of all RNS´s equates.

## Solving multiple order problems

* **Direct products of net abstraction relation** enable simultaneous observation in the power sets of the nets.
* The review of the achieved upper limit for the cardinality of the set of the possible solutions reveals the character of *undecidability, even though it will be proven* that in all evolution levels class solutions in transformation processes inevitably drop into some abstract class thus quarantining a *closure system*.
* **Multiple order net class rewriting systems** bring new depth for observation in problem structure layers spotlighting new interfaces of the system evolution levels.
  + Multiple order *parallel quotient relations* coinciding relevant algebra enable an expansion of already found solutions to enter into further conceptual levels by altering solution space to desired level by *multiple order transducers*.

### Finally by iterating multiple TD transformations in evolution chains

unlimited solving structure will be achieved - so utterly important considering **self-**

### evolving unrestricted solving processes.

***Conclusions***

The reached theory serves as the *fundamental algorithmic tools for independency* of solving systems in robotics and overall in artifacts. The study represents a new way to *describe knowledge with generalized universal algebra allowing loop structures* so very important in AI languages and which gives an extensive variety of *notional relations between net entities* without restricting the semantic use. Consequently a new syntax model for solving problems defined by said nets is established flexibly utilizing notional similarities with original problems to further *match solutions in memory data banks* thus additionally creating transducer graphs of solving rewrite systems and thereof *closure system of solving classes*. The study introduces *universal partitioning to widen environmental attachments* subject to abstraction relations, yielding *universal macros* from parallel TD-solutions. Net *NUO-presentations* are delivered providing more general coverage enabling *net block homomorphism* to be used for *TD-solution generation*. A special attention is given to cardinalities of basic solutions. Second order parallel relation is introduced for distinct solution set bases. Finally *direct products of power sets in abstraction relations* serve as ingredients for *multiple level abstraction* algebra which is taken in account for determining *self-evolving solving* systems. This is reached by tree different stages offering combinational approach in multiple power solution families and iterative solving, thus creating solution basis for evolutional levels.